

# Estimating the Elasticity of Substitution between Capital and Labour

Julija Harrasova, University of Edinburgh

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## Abstract

This study estimates the elasticity of substitution between capital and labour as well as rates of factor-augmenting technical progress across both the aggregate economy and individual industries in the UK and Scotland. Both single equation and system estimation frameworks are used and the finding is that elasticity estimates are highly sensitive to the choice of method. Using system estimation techniques previously not applied to the UK, the finding is that aggregate elasticity is 0.94, which is significantly higher than previous results for the UK. Also, sectoral elasticities are characterised by high variability. Technological progress is also found to be overall net labour-augmenting which supports the neoclassical steady-state growth theorem. With augmentation directed at labour, and under complementarity between factors of production, the conclusion is that technical change is capital-biased, which is consistent with the declining labour share of income observed in UK data. Aggregate elasticity in Scotland is 1.3, however, this result should be interpreted with caution as it suffers from small sample bias and may reflect poorer quality of data.

## 1. Introduction

Since the introduction of the Constant Elasticity of Substitution (CES) production function by Arrow et al. (1961), many studies have attempted to estimate the elasticity of substitution between capital and labour. However, limited consensus has been reached with regards to value of elasticity in advanced economies. Most recently, the bulk of empirical evidence seems to suggest elasticity below unity in the U.S. (Chirinko et al., 2004; Klump et al., 2007; Herrendorf et al., 2015), while others argue that the observed decline in labour share of income is only consistent with elasticity above unity (Karabarbounis and Neiman, 2013; Piketty and Zucman, 2014).

The primary objective of this dissertation is to provide an up-to-date estimate of the elasticity of substitution between capital and labour for use in Computable General

Equilibrium (CGE) models of Scotland or the United Kingdom. A CGE model is used to assess the impact of policy changes and shocks on the wider economy. Elasticity is one of the parameters that is determined exogenously in the model and is typically sourced from an econometric estimate. The current elasticity of substitution used in the Scottish Government's CGE model is set to 0.3 and is applied uniformly across all sectors within the model (Figus et al., 2017; Harris, 1989).

Therefore, this dissertation attempts to answer the following questions:

- 1) What is the value of aggregate elasticity of substitution between capital and labour in the UK?
- 2) How much does elasticity differ at industry and sectoral level?
- 3) Is technology neutral or factor augmenting in the CES production function, and what is the bias of technical change?

I start by reviewing the extensive literature available for elasticity estimation and discuss the merits and limitations of various estimation methods. Evidence for the UK seems to suggest an aggregate elasticity in the range of 0.4. More recently, system estimation approaches have gained more popularity and when applied to US data indicate a higher degree of substitutability. A contribution of this study is to enhance understanding of substitutability of capital and labour across the aggregate economy and at industry level in the UK using a system estimation approach. To the best of my knowledge, there is no recent study that estimates elasticity of substitution between capital and labour and rates of factor-augmenting technical change across industries for the UK or Scotland.

Several sets of estimates of aggregate and industry elasticities are produced, based on different estimation methods and alternative restrictions on the parameters of the production function. I start by presenting the UK results from the single equation estimates using OLS and First Difference methods. The estimated aggregate elasticity is in the neighbourhood of 0.5. The estimates are further improved by using system estimation techniques that account for cross-equation correlation in disturbances and endogeneity of the regressors. Using this approach, elasticity rises to 0.80-1.06 when using Nonlinear Seemingly Unrelated Regression method and to 0.94 in 3SLS. I also provide sectoral estimates for Scotland and find that aggregate elasticity is somewhat higher (1.3).

However, this result should be interpreted with caution as it suffers from small sample bias and may reflect poorer quality of data.

The dissertation is organised as follows: section II provides literature review, section III presents the theoretical framework and model, section IV covers data sources and assumptions, section V discusses the econometric approach, section VI presents results and discussion, and section VII concludes.

## **2. Literature Review**

### **2.1 The Role Of Elasticity**

The elasticity of substitution ( $\sigma$ ) is an important concept in many areas of economic theory. In production function, it is a parameter that measures how substitutable are labour and capital when their prices change keeping the level of output fixed. The parameter was first introduced by Hicks (1932) in a two-input production function.

The CES production function nests multiple forms of technology depending on the value of elasticity. For example, the case of well-known Cobb- Douglas production function assumes unitary elasticity of substitution between labour and capital. This functional form also implies constant capital (and labour) share of income, which is also known as a Kaldor fact (Kaldor, 1957). Recently, however, the labour share has declined in the advanced economies (Elsby, 2013). Piketty and Zucman (2014) and Karabarbounis and Neiman (2013) link this to above unitary value of elasticity of substitution.  $\sigma > 1$  implies that capital and labour are easily substitutable and therefore, when relative price of either factor of production changes, firms substitute away from a more expensive input towards a cheaper one. This in turn increases the intensity of the cheaper input in the production and raises the share of total income that goes to that input. Therefore, elasticity value is important for understanding trends in the distribution of income between factors of production.

The motivation behind a large chunk of empirical literature on estimating  $\sigma$  has been precisely to determine whether the value is below or above unity, or exactly unity. The consensus around the value of elasticity is clearly necessary for the understanding of the evolution of income shares and many theoretical applications. It is also important for applied work, such as economic modelling, where results may be sensitive to values of parameters.

The concept of elasticity is closely related to other parameters of production function such as efficiency. The neoclassical steady-state growth theorem, as defined by Uzawa (1961), requires that for an economy experiencing the steady-state growth the technical progress must be labour-augmenting or the production function must be Cobb-Douglas.<sup>1</sup> The assumption of the direction of augmentation is rather restrictive. Acemoglu (2002), for example, argues that there is a possibility of transitional capital-augmenting technical growth and with  $\sigma < 1$  the steady state growth is only purely labour augmenting. Therefore, estimation of the production function can verify whether long-run data supports the assumptions of the steady state neoclassical growth.

The additional focus of this dissertation is to estimate  $\sigma$  across industries. Some heterogeneity is expected across different sectors of the economy which can be indirectly observed from differences in factor intensities. For example in the UK, the average annual growth rate of capital intensity (K/L) between 1971-2005 has been 3.3% in agriculture, 2.7% in manufacturing, 4.7% in financial, real estate and business services.

## 2.2 Empirical Estimations

With a wealth of empirical literature available, I distinguish between different estimation methods used for identifying  $\sigma$ . The majority of studies can be classified as using either *direct* (i.e., estimation of the production function equation directly either using a linear approximation or applying nonlinear least squares) or *indirect* estimation methods (i.e., identifying elasticity from related equations), or a combination of both.

Notwithstanding differences in approaches, the lack of consensus in elasticity estimation has been further amplified by practical data problems such as outliers, serial correlation, measurement issues, and structural breaks. The choice of data also seems to be affecting the estimated  $\sigma$  ranges with crosssection estimates generally reporting higher elasticity than the time series regressions (Berndt, 1976).

I start to review the existing literature by looking at the indirect methods first, as these were prevalent in the initial studies, and then move onto direct methods. Lastly, I discuss the system estimation approach that combines the elements of both.

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<sup>1</sup> As part of the theorem, for the capital-output ratio to remain constant and for output per worker to grow at a constant rate, the technical growth must be labour-augmenting (Jones and Scrimgeour, 2005).

### 2.2.1 Indirect Methods

#### First Order Conditions

The most notable study at this front is that of Berndt's (1976) where the log-linearised first order conditions with respect to inputs of production and combinations thereof are used to identify the elasticity. Berndt(1976) assumes Hicks-neutral technical progress and finds that estimates of  $\sigma$  are insignificantly different from unity (i.e. supporting the Cobb-Douglas case). It is worth noting, however, that assumptions about technical change can affect the identification of elasticity. Antras (2004) re-examines Berndt's (1976) approach and shows how Hicks-neutral technical change biases the estimated elasticity towards unity. Antras(2004) himself re-estimates the similar equations allowing for factor-augmenting change and finds the range of elasticity estimates that are significantly below unity.

Estimations based on the first order conditions, however, are likely to suffer from simultaneity bias due to factor demands depending on the relative factor prices which in turn again depend on factor demands. A solution to this is to use truly exogenous instruments that are uncorrelated with the error term but are correlated with the regressors.

#### User Cost of Capital

Another strand of the literature attempts to estimate  $\sigma$  using the user cost of capital approach.<sup>2</sup> In these models, the elasticity parameter can be identified from the equations that relate capital stock, investment and the user cost.

Chirinko et al. (2004) use cross-sectional variation and time-averaged firm-level data, and find that the elasticity in the region of 0.4 in the U.S. Ellis and Price (2004) estimate an equation relating the investment to the user cost of capital and find elasticity of 0.45. Barnes et al. (2008) use a time-averaging approach on the UK firm-level panel data and also find the elasticity of 0.4.

While estimates from above papers appear to be similar, use of the firmlevel data may

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<sup>2</sup> The user cost of capital takes into account an interest, depreciation and tax rates to reflect the true cost of owning an asset.

not fully capture the substitution that takes place at the industry level. In fact,  $\sigma$  is supplied at the sectoral level in the CGE model and therefore should reflect substitution at that level.

Moreover, the investment equation for this approach is derived from the capital first order condition. Simultaneity is also a concern here: user cost (the regressor) reflects the market interest rate which is determined through the equilibrium in the financial markets. A positive shock to the investment demand will push the interest rates up and since the latter is embedded in the user cost definition, there will be a positive correlation between the user cost and the error term of the investment equation.

## 2.2.2 Direct Estimation

### Linear Approximation (Kmenta)

As computational power of computers was weak in 1960s, the estimation of nonlinear in parameters of CES function was somewhat problematic. Kmenta (1967) has developed a linearised version of the CES function which was favoured due to its computational simplicity but came with a major limitation: the identification of elasticity is only possible under assumption of Hicks-neutral change. Performance of this estimation method is also questionable: Leon-Ledesma et al. (2010) show in Monte-Carlo setting that this approach under-estimates elasticity in small samples and only performs well when  $\sigma$  is close to 1.

### Nonlinear Estimation

Estimation of nonlinear equations comes with a number of challenges. A typical implementation of the nonlinear least squares (NLS) method uses the iterative algorithm search to minimise the objective function (i.e. the sum of squared residuals). But the objective function has large flat areas implying that a range of elasticities can satisfy a minimum (Harris, 1989). Henningsen and Henningsen (2012) highlight the following issues encountered with CES estimation using NLS:

- the SSR is non-smooth with extremely flat surfaces around the minimum point;
- the CES function (and the SSR) is discontinuous when  $\sigma \rightarrow 1$ ;

- presence of rounding errors around unitary elasticity leads to miscalculation of the predicted output which distorts calculation of the SSR.

Koesler and Schymura (2015) apply NLS to the CES function equation and use the pooled (by country) industry-level data for 1995-2007. The estimated elasticity is largely below unity. They also find no substantial variation in  $\sigma$  across regions, however, they are unable to control directly for the regions fixed effects and rather compare estimations for different regions to arrive at this conclusion (e.g. EU countries vs BRIC countries).

It does not seem unreasonable to assume that advanced economies, for example, such as the UK and the U.S. have somewhat similar production technologies and rates of technical progress. Nevertheless, there is lack of studies that estimate elasticities in a multi-country framework.

### **2.2.3** *System Estimation*

In recent years, a supply-side normalised system estimation has gained increased popularity among the researchers. The normalisation of the CES production functions has been developed by Klump and De La Grandville (2000). The main idea behind the normalisation is to fix the baseline values of key variables and express the function relative to that baseline. The factor-biased technical change can then be identified and technical parameters can be interpreted with respect to that baseline point (Klump et al. 2007). Since variables are measured in different units, indexing can also help to overcome dimensional issues (Steenkamp, 2017).

I summarise the empirical estimations of  $\sigma$  using the normalised system approach in the table 2.1. The influential paper by Klump et al. (2007) was a first empirical attempt to estimate elasticity using this approach. They used a nonlinear seemingly unrelated regression method to estimate jointly a system of three equations that combine the CES production function with the expressions of the factor income shares. Leon-Ledesma et al (2010) assess (Table 2.1) performance of various estimators in identifying elasticity, using a Monte- Carlo approach, and find that the system estimation is superior to single equation estimates.

To the best of my knowledge, the only industry-level studies of the aggregate elasticity

using the system approach are that of Young (2013), who estimates elasticities for the US industries, and Steenkamp (2017), who does the same for New-Zealand. Young (2013) finds that majority of industry estimates are below unity and when aggregated at sectoral level (agriculture, manufacturing, services) the elasticities lie within a rather narrow range (i.e. 0.46 - 0.68). Steenkamp (2017) finds large variability in industry estimates and gets above unitary elasticity for 8 out of 18 sectors.

### 3. The Model

#### 3.1 A General CES Production Function

I start by assuming that a production at industry-level can be characterised by a production function with constant elasticity of substitution (CES). The general form of a two-input CES production technology in industry  $j$  is defined as follows:

$$Y_j = F(A_j^K K_j, A_j^L L_j) = [\alpha_j (A_j^K K_j)^{\frac{\sigma_j-1}{\sigma_j}} + (1 - \alpha_j) (A_j^L L_j)^{\frac{\sigma_j-1}{\sigma_j}}]^{\frac{\sigma_j}{\sigma_j-1}} \quad (3.1)$$

where  $Y_j$  is real output in industry  $j$ ,  $K_j$  and  $L_j$  are capital and labour

inputs,  $A_K$  is capital-augmenting and  $A_L$  is labour-augmenting technological change, and  $\sigma_j \in [0, \infty)$  is the elasticity of substitution between capital and labour in the industry  $j$ , and  $\alpha_j$  is a distribution parameter<sup>3</sup> that captures the relative importance of each input in the production process.<sup>3</sup> Elasticity of Substitution and Technical Change The elasticity of substitution is given by the proportionate change in the ratio of factors of production due to a change in the marginal products:

$$\sigma = \frac{d \ln\left(\frac{K}{L}\right)}{d \ln\left(\frac{F_L}{F_K}\right)} \quad (3.2)$$

Consequently, under assumption of competitive factor markets the marginal products are equal to factor prices and  $\sigma$  measures how much substitution takes place when one input becomes relatively more expensive, holding output fixed. Labour and capital are gross

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<sup>3</sup> With perfectly-competitive markets,  $\alpha_j$  is equal to the capital share of income (Klump et al., 2007).

substitutes in production process when  $\sigma > 1$ , and gross complements when  $\sigma < 1$  (Acemoglu, 2002).

Technological progress can enter the production function in various ways. I include factor-augmenting technology and assume that each technology parameter grows at constant rates  $\gamma_K$  and  $\gamma_L$ :

$$A^K = A_0^K e^{\gamma_K t} \quad (3.3)$$

$$A^L = A_0^L e^{\gamma_L t} \quad (3.4)$$

Labour-augmenting (or Harrod-neutral) technological progress has an impact on output equivalent to introducing more labour input into production; capital-augmenting (or Solow-neutral) progress is equivalent to having more capital input. It is possible that technological progress affects labour and capital in a symmetric way (i.e. Hicks-neutral change). The nature of technical change together with value of  $\sigma$  has an important implication for the bias of technological change. The direction of the bias determines which factor's compensation increases in response to technical progress.

I follow Acemoglu(2002) and show how the bias depends on  $\sigma$  through the ratio of the marginal products. The relative marginal product of capital (MPK) can be defined as:

$$\frac{F_K}{F_L} = \frac{\alpha}{1 - \alpha} \left( \frac{A_0^K e^{\gamma_K t}}{A_0^L e^{\gamma_L t}} \right)^{\frac{\sigma-1}{\sigma}} \left( \frac{K}{L} \right)^{-\frac{1}{\sigma}} \quad (3.5)$$

With  $\sigma > 1$ , net capital-augmenting technical change ( $\gamma_K > \gamma_L$ )

increases the relative MPK, while net labour-augmenting change ( $\gamma_L > \gamma_K$ ) decreases it.

- With  $\sigma < 1$ , the capital-augmenting change decreases the relative MPK, while labour-augmenting change increases it.
- With  $\sigma = 1$  (or in Cobb-Douglas case), the relative MPK is independent of  $A$  and technical change is not biased.

Therefore, when technological change is labour-augmenting, and inputs are gross-substitutes, it is also labour-biased. Conversely, when inputs are gross-complements,

labour-augmenting change is capital-biased. Intuitively, with complementary of capital and labour, increase in  $A^L$  rises the demand for  $K$  by more than the demand for  $L$  (recall that labour-augmenting progress is equivalent to having more labour input, so with more productive *effective* labour more capital is needed) and the relative MPK rises. This in turn results in larger share of income attributed to capital.

Profit Maximisation

Under perfectly competitive product and factor markets, the profit maximisation of a representative firm implies two first-order conditions, equating marginal products to factor prices:

$$P_t[\alpha_j(A_t^K K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t^L L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}-1}(1-\alpha_j)(A_t^L)^{\frac{\sigma-1}{\sigma}} L_t^{\frac{\sigma-1}{\sigma}-1} - W_t = 0$$

$$\left[\frac{W_t}{P_t}\right]^\sigma = \frac{Y_t}{L_t}(1-\alpha)^\sigma (A_t^L)^{\sigma-1}$$

Substituting (3.4):

$$\left[\frac{W_t}{P_t}\right]^\sigma = \frac{Y_t}{L_t}(1-\alpha)^\sigma (A_0^L e^{\gamma L t})^{\sigma-1} \quad (3.6)$$

$$P_t[\alpha(A_t^K K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t^L L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}-1} \alpha (A_t^K)^{\frac{\sigma-1}{\sigma}} K_t^{\frac{\sigma-1}{\sigma}-1} - R_t = 0$$

$$\left[\frac{R_t}{P_t}\right]^\sigma = \frac{Y_t}{K_t}(\alpha)^\sigma (A_t^K)^{\sigma-1}$$

Substituting (3.3):

$$\left[\frac{R_t}{P_t}\right]^\sigma = \frac{Y_t}{K_t}(\alpha)^\sigma (A_0^K e^{\gamma k t})^{\sigma-1} \quad (3.7)$$

### 3.2 Normalised System

I follow a range of papers that estimated CES function for the US and normalise the production function such that all variables are expressed relative to the benchmark point. Klump et al. (2007) suggest that the baseline point should be calculated from the data and set equal to sample average, as it removes the short-term fluctuations from the data. Therefore, I calculate  $K$ ,  $L$  and  $Y$  as geometric sample means of capital, labour and value

added variables, and  $\bar{t}\bar{\alpha}$  as arithmetic means of time and capital share.<sup>44</sup> At this common baseline point, the capital share of income is not biased by the growth in efficiency parameters (and therefore not influenced by the direction of augmentation) but is just equal to the distribution parameter  $\alpha$  which measures relative importance of capital in the production function (Klump et al., 2007).

I follow Herrendorf et al. (2015) and express the system consisting of the normalised CES production function:

$$Y_t = \bar{Y} \left[ \bar{\alpha} \left( A_0^K e^{\gamma_K(t-\bar{t})} \frac{K_t}{\bar{K}} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \bar{\alpha}) \left( A_0^L e^{\gamma_L(t-\bar{t})} \frac{L_t}{\bar{L}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (3.8)$$

The associated first order condition expressions for the above production function are:

$$\frac{W_t}{P_t} = \frac{(1 - \bar{\alpha})\bar{Y}}{\bar{L}} e^{\gamma_L(t-\bar{t})\left(\frac{\sigma-1}{\sigma}\right)} \left( \frac{Y_t}{\bar{Y}} \frac{\bar{L}}{L_t} \right)^{\frac{1}{\sigma}} \quad (3.9)$$

$$\frac{R_t}{P_t} = \frac{\bar{\alpha}\bar{Y}}{\bar{K}} e^{\gamma_K(t-\bar{t})\left(\frac{\sigma-1}{\sigma}\right)} \left( \frac{Y_t}{\bar{Y}} \frac{\bar{K}}{K_t} \right)^{\frac{1}{\sigma}} \quad (3.10)$$

Above equations represent a supply-side system normalised around the benchmark point which can now be estimated by imposing cross-equation restrictions.

## 4. Data and Sources

### 4.1 United Kingdom

Data for the UK is obtained from the EU KLEMS 2009 release (EU KLEMS, 2009), as it contains the longest time-series covering the UK between 1970 to 2007.<sup>5</sup> The list of

<sup>4</sup> use geometric means for variables that grow over time as arithmetic means will skew the average upward.

<sup>5</sup> EU KLEMS is an industry level, growth and productivity research project that ran from 2003 until 2008 and was funded by the European Commission.

variables obtained from the database is displayed in Table 4.1: (Table 4.1)

To estimate a two-input production function, I use gross value added (VA) as a measure of output: it excludes intermediate inputs and represents the value of labour and capital used in production. I use total hours worked by persons engaged (H EMP) as a measure of labour input and real fixed capital stock expressed in 1995 prices (K GFCF) as capital input.

I then transform the variables to account for price inflation and calculate additional variables required for the estimation. In what follows, I discuss the calculation approach for each variable presented in the Table 4.2. (Table 4.2)

I convert nominal VA and COMP time series into real variables by applying the value added price index (VA P). COMP variable excludes self-employed income. Whereas EU KLEMS make an adjustment to labour compensation by assuming that self-employed workers in each industry are earning same compensation per hour as employees, I find this assumption rather restrictive as it ignores possibility of self-employed workers earning higher (or lower) income in some industries. For example, Elsby et al. (2013) show how the "averaging" approach implies negative capital compensation for proprietors' income in 1980s and suggest that payroll share of income tracks most closely the developments in the overall labour share. In this data, such imputation also results in labour compensation exceeding the value added which in turn gives negative capital compensation for some years and industries. Moreover, evidence from the Family Resources Survey (ONS, 2018) for the UK shows that on average the level of earnings for self-employed workers is lower than for employees.<sup>6</sup> Without detailed data capturing distribution of earnings for self-employed vs employees across industries, I decide to use the employee compensation. The remaining variables are then calculated as per formulas presented in the table 4.2.

## 4.2 Scotland

Scottish industry-level production data is obtained from the Input-Output tables produced by the Scottish Government (2017). The time series are rather short and cover a period between 1998 and 2015. While it is possible to estimate a system with 3 equations and 3

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<sup>6</sup> Mean earnings of self-employed were around 240 a week and for employees - 400 a week in 2016.

parameters with this sample size, our estimates are likely to suffer from a small sample bias.

I use gross value added (GVA) as output measure and gross fixed capital formation (GFCF) as capital input. I convert the former into real series by applying the ONS Value Added Deflator (ONS, 2017). Ideally, production function should be modelled based on the levels of the capital stock, but no such data is available for Scotland. Therefore, I use GFCF data as capital input in the estimation. GFCF represents a component of the GDP when measured through the expenditure approach and captures net capital expenditure (acquisitions less disposals of assets). I then apply the ONS GFCF Deflator (ONS, 2016) to transform nominal values into real. Lastly, I construct

labour input following the approach suggested by the Scottish Government which is to obtain Full Time Equivalent values for industry-level employment using a multiplier approach.<sup>7</sup>

Similarly to the UK estimation, I use labour compensation data available in the Input-Output tables which is also converted into real values using the ONS Value Added deflator. Real capital compensation, factor prices and capital share of income are constructed in a similar way as detailed in the table 4.2 for the UK.

### Econometric Approach

I start with the estimation of individual equations using OLS and FD, and then move onto the system estimation by applying more efficient estimators such as NLSUR and 3SLS. The system approach is superior in economic sense as it captures both the production technology (the CES production function) and the optimising behaviour of firms (the First Order Conditions).

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<sup>7</sup> The summary of this calculation method is provided in the Appendix 2.

## 5.1 OLS and FD

### OLS

I take the natural logarithm of the First Order Conditions with respect to labour and capital, equations (3.9) and (3.10) respectively to get<sup>1</sup>:

$$\ln \frac{Y_t}{L_t} = \sigma \ln \frac{W_t}{P_t} + (1 - \sigma) \gamma_L t + (1 - \sigma) \ln A_0^L + \sigma \ln \frac{1}{(1 - \alpha)} + \epsilon_{lt} \quad (5.1)$$

<sup>1</sup> Note that a constant term in each equations is defined as  $\mu_l = (1 - \sigma) \ln A_0^L + \sigma \ln \frac{1}{(1 - \alpha)}$ ,  $\mu_k = (1 - \sigma) \ln A_0^K + \sigma \ln \frac{1}{\alpha}$  and  $\mu_{kl} = (1 - \sigma) \ln \frac{A_0^L}{A_0^K} + \sigma \ln \frac{1 - \alpha}{\alpha}$ ; and  $\epsilon_{lt}, \epsilon_{kt}$  and  $\epsilon_{klt}$  are the error terms in each equation.

$$\ln \frac{Y_t}{K_t} = \sigma \ln \frac{R_t}{P_t} + (1 - \sigma) \gamma_K t + (1 - \sigma) \ln A_0^K + \sigma \ln \frac{1}{\alpha} + \epsilon_{kt} \quad (5.2)$$

By subtracting (5.2) from (5.1) I get:

$$\ln \frac{K_t}{L_t} = \sigma \ln \frac{W_t}{R_t} + (1 - \sigma) (\gamma_L - \gamma_K) t + (1 - \sigma) \ln \frac{A_0^L}{A_0^K} + \sigma \ln \frac{1 - \alpha}{\alpha} + \epsilon_{klt} \quad (5.3)$$

Equations (5.1) – (5.3) can be estimated by OLS. Note that the technology parameters are not separately identifiable from the elasticity.

Above equations are expressed for the factor-augmenting technology, captured in parameters  $\gamma_K$  and  $\gamma_L$ , but the estimated coefficients are still interpretable under Hicks-neutral technology.<sup>2</sup>

### First-Difference Estimator

As it will turn out, the variables used in the estimation are largely non- stationary, so the natural solution is to apply first-difference transformation to the equations (5.1) – (5.3):

$$\Delta \ln \frac{Y_t}{L_t} = \sigma \Delta \ln \frac{W_t}{P_t} + (1 - \sigma) \gamma_L + \Delta \epsilon_{lt} \quad (5.4)$$

$$\Delta \ln \frac{Y_t}{K_t} = \sigma \Delta \ln \frac{R_t}{P_t} + (1 - \sigma) \gamma_K + \Delta \epsilon_{kt} \quad (5.5)$$

$$\Delta \ln \frac{K_t}{L_t} = \sigma \Delta \ln \frac{W_t}{R_t} + (1 - \sigma) (\gamma_L - \gamma_K) + \Delta \epsilon_{klt} \quad (5.6)$$

<sup>2</sup> With Hicks-neutral technology, the production function is defined as  $Y_t = A_0 e^{\gamma t} [\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$  and coefficient on  $t$  in the equations (5.1) and (5.2) changes to  $(1-\sigma)\gamma$ , and the coefficient on  $t$  in the combined equation (5.3) is now  $(1-\sigma)(\gamma-\gamma) = 0$ .

## 5.2 Nonlinear Seemingly Unrelated

### Regression

OLS is efficient and consistent estimator for individual equations but when contemporaneous errors are correlated across equations, greater efficiency can be achieved by estimating the system of equations jointly. Another advantage of a system estimation is that it allows to impose cross-equation restrictions on the parameters.

Zellner (1962) introduced Seemingly Unrelated Regression (SUR) as a way of estimating multiple equations that are related through the correlation between the error terms associated with each equation.

In linear form, I have a system of  $i = 1, \dots, M$  equations and  $t = 1, \dots, T$

observations<sup>8</sup>:

$$\mathbf{y}_1 = \mathbf{X}_1 \boldsymbol{\beta}_1 + \boldsymbol{\epsilon}_1 \quad (5.7)$$

$$\mathbf{y}_2 = \mathbf{X}_2 \boldsymbol{\beta}_2 + \boldsymbol{\epsilon}_2 \quad (5.8)$$

$$\mathbf{y}_M = \mathbf{X}_M \boldsymbol{\beta}_M + \boldsymbol{\epsilon}_M \quad (5.9)$$

I assume strict exogeneity of regressors,  $\mathbf{X}_i$ , and homoskedasticity. I assume that

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<sup>8</sup> The model can be extended to a nonlinear setting:  $\mathbf{y}_i = \mathbf{g}_i(\boldsymbol{\beta}, \mathbf{X}) + \boldsymbol{\epsilon}_i$

disturbances  $\epsilon_{it}$  are correlated across equations but uncorrelated across observations:

$$E[\epsilon_{it} \epsilon_{js} | \mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_3] = \sigma_{ij} \quad \text{if } t = s \text{ or } 0 \text{ otherwise}$$

The estimator for SUR is Feasible Generalised Least Squares (FGLS): first, each equation is estimated by the least squares and residuals are obtained to construct a consistent estimate of the variance-covariance matrix (VCE); the latter is then used to obtain an FGLS estimator:

$$\hat{\beta} = (\mathbf{X}'\hat{\Omega}^{-1}\mathbf{X})^{-1} \mathbf{X}'\hat{\Omega}^{-1}\mathbf{Y} \quad (5.10)$$

where  $\hat{\Omega}^{-1}$  is the VCE.

I also choose an option to perform an iterative procedure, i.e. after FGLS estimation I obtain new residuals which are then used to re-estimate the VCE and obtain the new FGLS estimator. The iterations stop until convergence is achieved: the elements of the VCE or parameters vector stop changing (or relative changes are small enough).<sup>9</sup>

I take a natural logarithm of the normalised system as defined in equations (3.8) - (3.10):

$$\ln\left(\frac{Y_t}{\bar{Y}}\right) = \frac{\sigma}{\sigma-1} \ln \left[ \bar{\alpha} \left( e^{\gamma_K(t-\bar{t})} \frac{K_t}{\bar{K}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\alpha}) \left( e^{\gamma_L(t-\bar{t})} \frac{L_t}{\bar{L}} \right)^{\frac{\sigma-1}{\sigma}} \right] + \epsilon_{yt} \quad (5.11)$$

$$\ln(w_t) = \ln\left(\frac{(1-\bar{\alpha})\bar{Y}}{\bar{L}}\right) + \frac{\sigma-1}{\sigma} [\gamma_L(t-\bar{t})] + \frac{1}{\sigma} \ln\left(\frac{Y_t \bar{L}}{\bar{Y} L_t}\right) + \epsilon_{lt} \quad (5.12)$$

where  $\delta_1 = \ln\left(\frac{(1-\bar{\alpha})\bar{Y}}{\bar{L}}\right)$  and  $w_t = \frac{W_t}{P_t}$

$$\ln(r_t) = \ln\left(\frac{\bar{\alpha}\bar{Y}}{\bar{K}}\right) + \frac{\sigma-1}{\sigma} [\gamma_K(t-\bar{t})] + \frac{1}{\sigma} \ln\left(\frac{Y_t \bar{K}}{\bar{Y} K_t}\right) + \epsilon_{kt} \quad (5.13)$$

where  $\delta_2 = \ln\left(\frac{\bar{\alpha}\bar{Y}}{\bar{K}}\right)$  and  $r_t = \frac{R_t}{P_t}$  The system can be estimated jointly with

parameter homogeneity restriction across equations. I set  $A_0^K = A_0^L = 1$ .

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<sup>9</sup> Another advantage of using the iterative procedure is that NLSUR is equivalent to the maximum likelihood estimator (Poi, 2008)

### 5.3 Three Stage Least Squares

To account for potential endogeneity of the regressors, I also estimate the normalised system in (5.11) - (5.13) using the Three Stage Least Squares (3SLS) estimator.<sup>10</sup> Endogeneity is almost inevitable in the production function framework: factor inputs are likely to be correlated with the unobservables; simultaneity may bias NLSUR results - by applying instruments, the robustness of these estimates can be verified.

3SLS is essentially a combination of two-stage least squares (2SLS) and SUR which in this context represents the third stage: first, the endogenous right-hand side variables are regressed on the instruments to obtain the "instrumented values"; then a nonlinear least squares regression is run with the instruments as the right-hand side variables; the last stage is similar to the step described in the SUR section: an estimate of the VCE is constructed using the residuals from the second stage, so that an FGNLS estimator can be obtained.

Following Klump et al.(2007), Leon-Ledesma et al.(2015) and Herrendorf et al.(2015), I use first lags of all variables, constant and a time trend as instruments to deal with endogeneity of regressors. This approach also deals with serial correlation in the error term which otherwise results in invalid standard errors. I follow Herrendorf et al.(2015) and add AR(1) structure to the error terms of the system equations which are then estimated by 3SLS using the Cochrane-Orcutt correction procedure.<sup>11</sup>

The instruments should be relevant (i.e. correlated with the endogenous regressors) and exogenous (i.e. not correlated with the error term in each equation). The relevance requirement is satisfied as our variables follow autoregressive process and  $x_t$  is correlated with  $x_{t-1}$ . Now we require exogeneity to be satisfied:

$$\mathbf{E}(x_{kt-1} \epsilon_{it}) = 0$$

The error term in i-th equation also follows AR(1) process with an i.i.d. disturbance:

$$\epsilon_{it} = \rho_i \epsilon_{it-1} + v_{it}$$

---

<sup>10</sup> use the 3SLS estimator in Eviews as Stata has only implementation for a linear system of equations.

<sup>11</sup> Cochrane-Orcutt method first estimates the autoregressive parameter  $\hat{\rho}$  and then transforms the estimation equation to account for AR(h) process in the error term (Verbeek, 2005).

where  $\rho_i$  is an autocorrelation coefficient such that  $|\rho_i| < 1$  and  $v_{it}$  is an i.i.d. disturbance. Since  $\rho_i$  can be estimated and  $v_{it}$  has a zero mean (so  $\mathbf{E}(x_{kt-1}v_{it}) = 0$  by construction), the lagged values of our regressors are exogenous.

## 6. Results and Discussion

I present the estimation results for the aggregate economy first to ensure comparability with previous studies. Industry-level empirical estimations of the elasticity of substitution between capital and labour are less common. After presenting industry results for the UK, I also include results from the Scottish estimation and discuss implications for the CGE model.

### 6.1 Individual Equations

Initially, the OLS regressions are estimated for the equations (5.1) - (5.3) based on the UK data. I suspect that the time series are serially correlated which is confirmed through the initial tests. The reported Durbin-Watson statistic for the OLS estimation of the labour and capital equations is 0.56 and 0.26 and indicates presence of positive serial correlation. As autocorrelation invalidates standard errors, I use Newey-West variance covariance estimator which handles serial correlation up to the chosen lag length.<sup>12</sup> The first three columns of the table 6.1 report the estimated aggregate elasticity of substitution for the UK.  $\sigma$  is estimated as 0.52, 0.49 and 0.43 for each equation respectively. It has, however, very wide confidence intervals: e.g., in the labour equation the 95% confidence interval is [0.159,0.884]. Technical growth parameters are not separately identifiable but can be recovered from  $t$  and  $\sigma$ : the labour-augmenting growth ( $\gamma_L$ ) is positive 2% per annum and capital-augmenting growth ( $\gamma_K$ ) is negative 1.2% per annum. The term  $\gamma_L - \gamma_K$  captures the net augmentation of technology growth: positive or negative values of this term imply net labour or net capital augmenting growth. Our results indicate net labour augmenting growth of 5.4% per annum, which seems to support the prediction of the neoclassical growth theory discussed in the section 2.1. These results are somewhat similar to those

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<sup>12</sup> use a *varsoc* command in Stata to select the lag order for each equation based on the reported information criteria and likelihood ratio test.

obtained by Antras (2004) using the US data and similar equations: he finds  $\sigma$  in the range of 0.64 - 0.87, capital augmenting growth of -1.3% and labour augmenting growth of 1.9%.

If we were to assume Hicks-neutral technical progress, the estimated coefficients on the equations would not change. By setting  $\gamma_L = \gamma_K = \gamma$  in each equation, it seems evident that the labour and capital equations identify completely different growth rates. The negative  $\gamma$  in capital equation implies that efficiency on average declined over the sample period which seems improbable. This rather contradictory result may be due to the fact that assumption of Hicks-neutral technology fails to explain the trends observed in data. Moreover, with  $\gamma_L = \gamma_K = \gamma$  the time trend coefficient in the combined equation should not be significantly different from zero. But a 95% confidence interval for the term  $\gamma_L - \gamma_K$  clearly excludes zero: [0.019, 0.090]. It is therefore likely that factor-augmenting technology is more appropriate assumption for the data. equation respectively. It has, however, very wide confidence intervals: e.g., in the labour equation the 95% confidence interval is [0.159, 0.884]. Technical growth parameters are not separately identifiable but can be recovered from  $t$  and  $\sigma$ : the labour-augmenting growth ( $\gamma_L$ ) is positive 2% per annum and capital-augmenting growth ( $\gamma_K$ ) is negative 1.2% per annum. The term  $\gamma_L - \gamma_K$  captures the net augmentation of technology growth: positive or negative values of this term imply net labour or net capital augmenting growth. Our results indicate net labour augmenting growth of 5.4% per annum, which seems to support the prediction of the neoclassical growth theory discussed in the section 2.1. These results are somewhat similar to those obtained by Antras (2004) using the US data and similar equations: he finds  $\sigma$  in the range of 0.64 - 0.87, capital augmenting growth of -1.3% and labour augmenting growth of 1.9%.

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To save space, I report industry-level OLS estimates under assumption of factor-augmenting technology in Table 9.1 in the Appendix 9. The mean values of  $\sigma$  are 0.512 and 0.319 in the labour and capital equation respectively. The estimated  $\gamma_L$  has a mean value of 2.43%; and  $\mu$  is mostly negative with the mean value of -0.55%. The combined equation gives  $\sigma$  with a much lower mean value of 0.089. This equation also indicates the net labour augmenting growth of  $\gamma_L - \gamma_K = 3\%$  per annum.<sup>13</sup>

Finally, it is suspected that the time series may be non-stationary and the Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) tests are performed with the null hypothesis of a unit root present in the variables.<sup>314</sup> The ADF test indicates that an alternative hypothesis of trend-stationarity is only supported for  $\ln(\frac{y^t}{L})$  when including 3 lags in the testing regression and for  $\ln(\frac{K^t}{L})$  with 1 lag. The ADF and PP tests reject the null of non-stationarity  $t$  across all variables and conclude that series are I(1).

FD estimation is a natural solution to non-stationarity which can result in spurious regression. I present the FD estimates on equations (5.4) - (5.6) for the aggregate economy in the columns 3-6 of Table 6.1. The estimated  $\sigma$  is now slightly lower for the labour equation, moderately lower for the capital equation and insignificant for the combined equation. Similarly, the  $\mu$  is now slightly lower at 2% and  $\mu$  is less negative at -0.4%. Since  $\sigma$  is insignificant in the combined equation, a positive value on  $\gamma_L - \gamma_K$  should be interpreted with caution. These results match those observed by Young (2013) who gets  $\sigma_L$  of 0.82 and  $\sigma_K$  of 0.24, net labour augmenting growth from the similar equations using the US differenced data.

I then estimate FD regressions for each industry. To save space, the estimation results are reported in Table 9.3 in Appendix 1. The estimated elasticity is somewhat lower for the majority of industries in the labour and capital equations (the mean values are 0.46, and 0.24 respectively) and mostly insignificant (for 26 out of 37 industries at 5% level) in the combined equation. Technical parameters indicate net labour augmenting growth where  $\sigma$  is significant.

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<sup>13</sup> When excluding industry 31 with somewhat implausible estimate for  $\gamma_K$  of 275.0% growth per annum.

<sup>14</sup> PP test accounts for serial correlation. Table 9.2 in Appendix 1 provides results of these tests with trend including 0, 1 and 2 lags.

In general, it seems possible that the aggregate elasticity lies below unity (as suggested by previous evidence for the UK), but variability in industry estimates is not consistent with previous studies, e.g. Young (2013), Steenkamp (2017) and Koesler and Schymura (2015) find a range of  $\sigma$ s above 1. A possible explanation for this might be that estimation of differenced equations comes at the cost of losing valuable information that is contained in the levels. Instead of capturing the long-run relationship inherent in the production function, FD estimation method focuses on the short-term changes. These results therefore need to be interpreted with caution.

## 6.2 System Estimation

### Initial Values Approach

Nonlinear estimation requires the specification of initial values of the parameters. The system as defined in (5.11) - (5.13) has  $\sigma_0, \gamma_{K0}, \gamma_{L0}$  and  $\bar{\alpha}$ . I follow Leon-Ledesma et al (2015) and set  $\bar{\alpha}$  equal to its sample average - letting  $\bar{\alpha}$  vary did not significantly affect estimates and fixing the value upfront should help with convergence to the unique parameter vector. I also set  $\gamma_{K0}$  and  $\gamma_{L0}$  to zero as coefficients did not change when a system was estimated using a range of plausible initial values while keeping  $\sigma_0$  constant.<sup>15</sup> I set  $\sigma_0$  equal to the OLS estimates from the labour and capital equation.

In addition to OLS estimates, I also use values close to unity such as 0.98 and 1.02 (I cannot specify elasticity of 1 due to it appearing in the denominator); and, finally, I also include the initial values of 1.3 and 1.7. This approach helps to validate robustness of the estimated parameters.

### 6.2.1 Aggregate Results

#### NLSUR

Table 6.2 presents the aggregate estimation results for the UK using the NLSUR method. I

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<sup>15</sup> Plausible values for technological progress are 1-2 % (i.e. annual TFP growth rate in the UK). I also experiment with labour-augmenting ( $\gamma_{K0} = 0$  and  $\gamma_{L0} = 0.02$ ) and capital-augmenting ( $\gamma_{K0} = 0.02$  and  $\gamma_{L0} = 0$ ) growth but neither influence the results while keeping  $\sigma_0$  invariant.

estimate the system under both assumptions of the factor- augmenting and Hicks-neutral technical progress.<sup>516</sup> With the former, the estimated elasticity is 0.82 when OLS estimates for the labour and capital equation (0.52 and 0.49, respectively) are used for  $\sigma_0$ . The technical growth is net labour augmenting with negative  $\mu_K$  of 2.6% and positive  $\mu_L$  of 3.1%. As a sensitivity check, I also vary the starting values of  $\sigma$  and find that estimates are not always stable. When I use the initial elasticity value of 0.98, the results discussed above still hold. However, when I supply  $\sigma_0 = 1.02, 1.3, 1.7$ , the elasticity estimate changes to 1.06 and remains stable. The technology parameters are more sensitive: for  $\sigma_0 < 1$  the system converges to the net labour augmentation with plausible per annum growth rates; and with  $\sigma_0 > 1$  to the net capital augmenting growth with positive and relatively high  $\mu_K$  of 65% and negative  $\mu_L$  of 2.1%.

These findings were also observed by Klump et al. (2007) who found that low starting values of elasticity (e.g. 0.3) lead to estimates below unity (0.5 in their case) but when values are varied in search of global optimum, then it is achieved with elasticity estimate of 1 but with implausibly high capital augmenting rate of 81% and negative labour augmenting growth of 21%.

To decide which model represents a better fit, I choose the model that maximises the log-likelihood value.<sup>17</sup> Sensitivity to change in  $\sigma_0$  indicates that an optimum where the estimated elasticity is 0.82 may be just local. Similarly, when I run the same estimation under the assumption of Hicks- neutral technical progress (i.e.  $\mu_L = \mu_K$ ), the estimated elasticity is 1.37 and technical growth is of 1% per annum. Leon-Ledesma et al. (2015) also find that the estimated elasticity increases from 0.7 to 1 when allowing for neutral technical progress. Based on the log-likelihood, the model with the elasticity of 1.06 and augmenting technology is preferred. Contrary to Klump et al. (2007), the preferred model for the UK resulted in a rather implausible rate of net capital augmenting technical growth and the model with  $\sigma = 0.82$  should not be fully discarded.

At this point, I also perform a number of coefficient restriction tests: the null hypothesis of the unitary elasticity is rejected at 1% level; Hicks- neutrality restriction

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<sup>16</sup> The ADF test on the normalised variables fails to reject the null of unit root, however, variables are I(1) see Appendix 3 for more detail.

<sup>17</sup> Klump et al. (2007) pick a model that minimises the determinant of residual covariance matrix and discuss equivalence (in an opposite sense) of this approach to maximisation of the log-likelihood. Steenkamp (2017) and Stewart (2017) use log-likelihood to compare their models.

( $\gamma_L = \gamma_K$ ) is also rejected at 1% level. Rejection of Hicks-neutrality assumption is in agreement with the findings of previous section. I also report the associated test-statistics from the ADF test which lead me to conclude that residuals are stationary using 5% critical values with exception of output equation in the second model.<sup>18</sup>

### 3SLS

3SLS results are presented in the table 6.3 - I use same range of  $\sigma_0$  as in the NLSUR estimation. Under the assumption of factor-augmenting technology and using initial values that lie below 1,  $\sigma$  is estimated in the region of 0.936,  $\gamma_K$  is negative 5.1% per annum and  $\gamma_L$  is estimated at 4% per annum.

The estimates are also sensitive to initial values: when  $\sigma_0 > 1$ , the elasticity estimate is exactly 1.00,  $\gamma_K$  is estimated at an implausible 99% per annum and  $\gamma_L = -55\%$  per annum. Since Eviews does not report the associated log-likelihood, I follow Klump et al.(2007) and Herrendorf et al.(2015) and select the model that minimises the determinant of the residual covariance matrix.<sup>19</sup> With this approach, the first model with  $\hat{\sigma} = 0.936$  is preferred as it has a lower determinant value. It also results in estimates that are slightly higher in absolute values compared to those found by Herrendorf et al. (2015) for US data using 3SLS estimation.<sup>20</sup>

Unitary elasticity and Hicks-neutrality restrictions are comfortably rejected as for previous estimations (see bottom rows of table 6.3). The estimated coefficients on AR(1) terms are 0.74, 0.78 and 0.99 for the capital, labour and output equations respectively. The residuals from each equation are plotted in Figure 6.1. As outlined in the methodology for 3SLS estimator, the instruments should be uncorrelated with the contemporaneous error terms and this is satisfied through a zero-mean disturbance  $v_t$ . I can test whether the residuals are white noise. I follow Herrendorf et al.(2015) and report the Portmanteau test results for the lags 1 through 5 and associated Q-stats in Table 9.9 (Appendix 3). The null hypothesis of no residual autocorrelation is not rejected at 5% level of significance.

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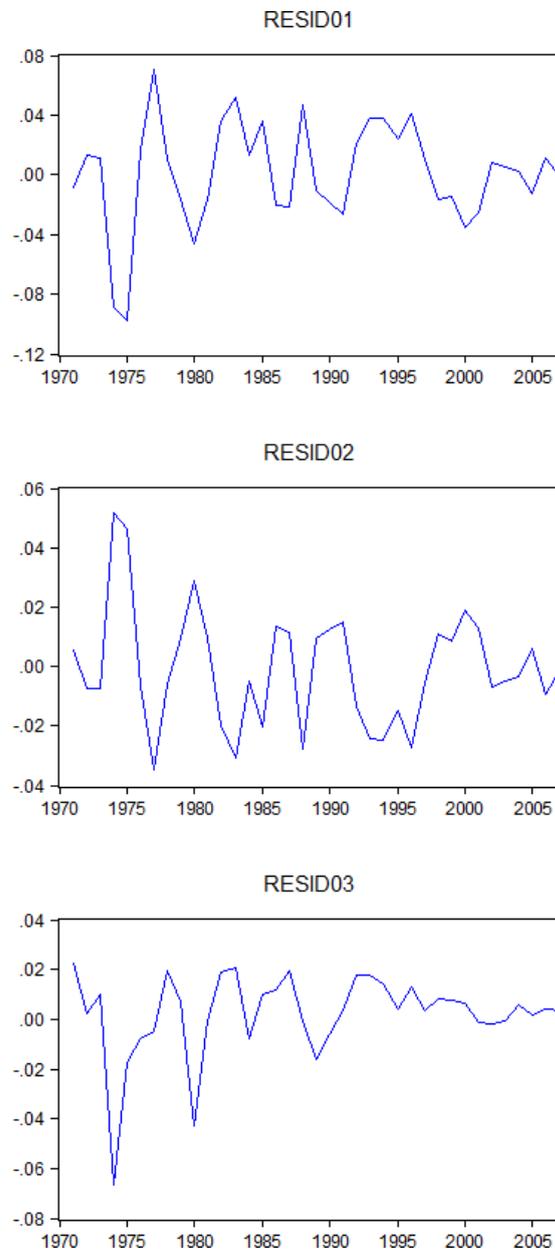
<sup>18</sup> I'm unable to perform a proper co-integration test for this nonlinear system and follow Leon-Ledesma et al.(2015) and Herrendorf et al.(2015) who estimate a similar system and check stationarity of residuals by performing an ADF test.

<sup>19</sup> The determinant of the residual covariance matrix should be close to 0 for an efficient estimation. If the errors are getting smaller, then the determinant also becomes smaller (Benchimol, 2013).

<sup>20</sup> Herrendorf et al. (2015) report an estimate of aggregate  $\sigma$  as 0.84 and find  $\gamma_K$  of -1% and  $\gamma_L$  of 2.2%.

**Figure 6.1:** Residuals from 3SLS estimation

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01, 02 and 03 represent residuals from the wage, rate and output equations respectively.

### 6.2.2 Industry Results

The NLSUR industry-level results are presented in columns 4-6 of the Table 9.5 (to save

space results are presented in the Appendix 9).<sup>21</sup> The NLSUR method produces mean  $\sigma$  of 0.83 (when excluding the 3 digit outliers which I discuss below) which is higher than the OLS results in the region of 0.5. The technological change parameters are estimated precisely.  $\gamma_K$  has a mean of -1% (when excluding the upper end outlier). Overall, technical growth is net labour augmenting with a mean value of 2.3% per annum. For some industries the estimated technical growth is net capital augmenting and these industries also tend to have elasticity that is either in the neighbourhood of 1 or exceeds it.

There are a few outliers in the estimated elasticities such as Education, Mining and Manufacturing of Non-Metallic Products which all produce implausibly high elasticities irrespective of the initial values. This may be reflective of the fact that the latter two industries have undergone substantial changes in the last decades with employment declining and production shifting abroad.<sup>22</sup> The estimate for industry 35 (Education) seems to reflect a poor fit of the model.

Columns 1-3 of Table 9.5 (Appendix 3) present 3SLS results for the industries. The mean value of elasticity is 0.81 (when excluding the Mining outlier) and elasticity estimates have less outliers. Technical growth is net labour augmenting when excluding the outliers:  $\gamma_K$  is -0.1% per annum and  $\gamma_L$  of 1.5% per annum.<sup>23</sup> As in NLSUR estimation, some industries have net capital augmenting growth and in some cases it is implausibly high, but these industries predominantly also have unitary or close to unitary elasticity which again indicates issues with identification at this point.

By looking at both NLSUR and 3SLS estimates and focussing on the aggregated sectors, I find that the primary sector (Agriculture, Hunting, Forestry and Fishing but excluding Mining) has elasticity in the range of 0.69-0.84. The Manufacturing sector has an elasticity in the range of 0.35- 0.57, Utilities 0.87-0.88, Construction 0.51-0.99, Trade 0.29-0.77, Hotels & Restaurants 0.46-1.39, Transport, Storage and Communication 0.55-1.53, Finance, Insurance, Real Estate and Business Services 0.31-0.57, Community, Social and

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<sup>21</sup> Only estimates for the best fit model are presented in the Table 9.5 - full estimation results when varying initial values are also available in the Appendix 3.

<sup>22</sup> For Mining sector, I also split the sample based on the point of structural break in early 1990s (based on  $K/L$  ratio). The estimated elasticity for 1970-1990 is 2.3 and for 1991-2007 is 1.37. While these results indicate a shift in substitutability over time, the sample size may introduce a bias and not much weight should be placed on these findings.

<sup>23</sup> Growth rates of above |20%| per annum are excluded.

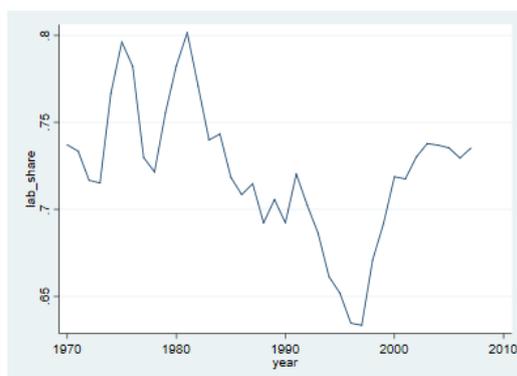
Personal Services 0.62-0.99.

The sectoral estimates found in this study are mostly consistent with previous empirical estimations. For example, Raval (2018) uses cross-sectional data on US manufacturing plants and estimates the elasticity of substitution between 0.3 and 0.5. Young (2013) and Herrendorf et al. (2015) estimate the supply-side system for the US and find elasticity in the manufacturing as 0.80 and 0.57-.69 respectively. Steenkamp (2017) finds elasticity of 0.49 in New-Zealand's manufacturing. My agriculture estimate is to some extent similar to that of Young's (2013) and Steenkamp's (2018) who both find  $\sigma$  of 0.68. In contrast, Herrendorf et al.(2015) find a somewhat higher elasticity of 1.58. It is difficult to compare service sector estimates due to differences in classification but estimates are somewhat lower as is expected in sector which produces mostly intangible goods.

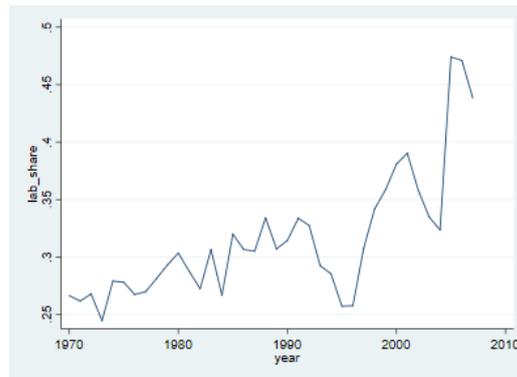
Interestingly, technical change is overall net labour augmenting in the manufacturing sector but at the same time, as per equation (3.5), it is also capital-biased due to gross-complementarity between inputs. This result is consistent with the decline in labour share of income the manufacturing as show in Figure 6.3.

In contrast, the technical change in agriculture is capital-augmenting and with  $\sigma < 1$  it is also labour-biased which should result in rising labour share. Finally, the aggregate economy is characterised by labour-augmenting and therefore capital-biased technical change. In other words, when labour becomes more productive, the marginal product of capital rises by more and in turn reduces the labour income share.

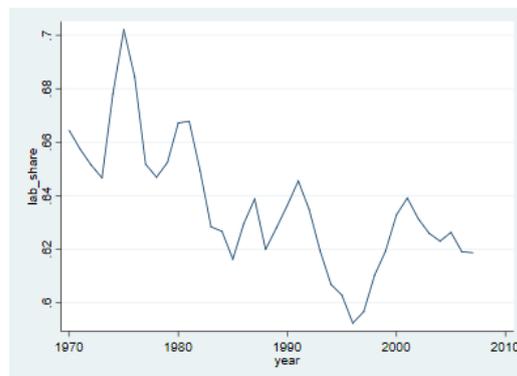
**Figure 6.3:** Labour share of income in the UK



(a) Manufacturing



(b) Agriculture



(c) Aggregate

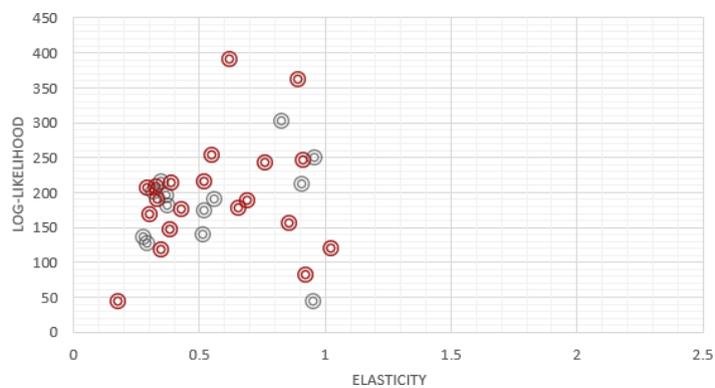
**Stability**

As was found in the aggregate results, the parameters are sensitive to the choice of initial values. For NLSUR and 3SLS estimations respectively, 13 and 16 industries converge to the unique stable vector of parameters indicating a global optimum. I only consider the estimates that either (a) remain stable across all initial values; or (b) may have changed once or twice but revert back to the original global optimum when varying the initial values. Further breakdown of industries is provided in table 9.10 in Appendix 4. I review, case by case, industries where values are sensitive to variation of  $\sigma_0$ . The common pattern across these industries is that when  $\sigma_0$  is sourced from the OLS results, which are largely below unity, the estimated elasticity across 3SLS and NLSUR also tends to be below unity. The top

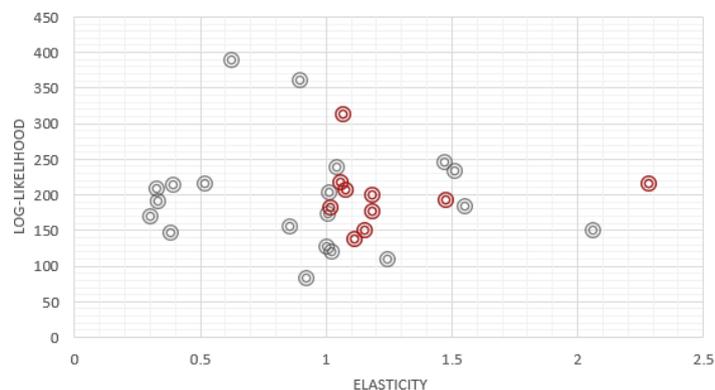
graph in the Figure 6.4 shows the NLSUR estimation log-likelihood values and associated elasticity estimates for  $\sigma_0 < 1$  across all industries (excluding high outliers). The estimates are predominantly  $< 1$  and some represent the global optimum. However, when  $\sigma_0 > 1$  (the bottom graph), then a number of industries cluster in the neighbourhood of 1 and a large share exceed one - some of these represent a global optimum too.

Klump et al. (2007) find similar results when varying initial values and relate poor tracking properties to the singularity of the system around  $\sigma = 1$ , and possibility of multiple optima above and below that point. This behaviour is likely to be explained by discontinuity of the CES production function around  $\sigma = 1$ . If the function is not continuous, the global maximum may not be attainable. A potential solution to this, as suggested by Henningsen and Henningsen (2012) for estimation of CES function using nonlinear least squares, is to adjust the optimisation algorithm (i.e. minimisation of the sum of squared residuals for NLS) such that values of output are approximated with their limits when  $\sigma \rightarrow 1$ .

**Figure 6.4:** Log-likelihood plot for the NLSUR estimation



$\sigma_0 < 1$



$\sigma_0 > 1$

This sample excludes outliers with a 3 digit elasticity. Red colour indicates a global maximum.

### **Performance of Estimators and Limitations**

Looking first at individual equations, I find that estimates from the first order condition with respect to labour are higher. Berndt (1976), Klump et al (2007) and Leon-Ledesma et al (2010) also find similar asymmetry in estimates. Berndt (1991) suggests that  $\sigma$  from the capital equation is systematically lower due to different speed of adjustment between factors of production. Since capital is fixed in the short run, less substitution is expected when relative prices change.

FD estimation produces similar estimates to OLS in at least half of the industries when estimating the labour equation and at least for a third when estimating the capital equation. The FD method resolves the nonstationarity problem but underestimates the elasticity of substitution as it focuses on the short run changes and does not build on the information contained in the levels of variables.

In addition, the OLS method is likely to produce downward-biased estimates of  $\sigma$ . First order conditions are supposed to reflect the long-run relationship between factors of production and their prices, but in the short run firms face frictions and therefore substitution is expected to be lower. Another source of potential bias, and the primary reason why I run the 3SLS estimation, is simultaneity in the individual equations.

It seems evident that elasticity produced by a system estimation is overall higher as it alleviates simultaneity bias inherent in single equations. These estimates are also more efficient and are characterised by narrower confidence intervals. The approach, however, has some limitations. The most notable one is sensitivity to the varying initial values of the parameters and possibility of multiple optima. Even though sensitivity of estimates is primarily around unitary value, a more thorough grid search across a wider range of initial values may help with identification of global optima.

In addition, all estimation methods are reflective of the quality of the data used. For example, labour input as measured in hours may ignore differences in quality of labour associated with different workers which allows correct identification of the labour-

augmenting productivity growth. A relevant adjustment of labour compensation for self-employment income is also required to correctly calculate labour share.<sup>24</sup> In addition, the rental rate of capital in this estimation is derived from capital compensation and stock, while some studies use user cost of capital data. In short, better quality data should enhance identification of parameters but there is limited availability of such series at industry level.

## 6.3 Implications for Scotland

### Scottish Results

In this section I present the estimates of elasticity for the sectors as defined in the CGE model of the Scottish economy using both the UK EU KLEMS data and Scottish data. I re-aggregate EU KLEMS data where necessary to match CGE classification (see the mapping between EU KLEMS and CGE in Table

9.11 in Appendix 5). The results from NLSUR estimation for Scotland and the range of elasticity for the equivalent NLSUR and 3SLS UK estimates are presented in the table 6.4. The full results of Scottish estimation are presented in Table 9.12. A few sectors have converged to implausibly high estimates of elasticity (manufacture of electrical equipment, chemicals, utilities and other manufacturing). Due to differences in aggregation between the EU KLEMS database and the CGE model, no estimates are supplied for sectors Other Manufacturing and Information & Communication. The aggregate elasticity for Scotland is estimated as 1.295. While there seems to be some degree of consistency between the UK and Scotland, the estimates for Scotland appear somewhat higher and may be reflective of data limitations, therefore should be interpreted with caution. Therefore, for use in the CGE model it is recommendable to use only estimates from the UK results.

### Implications for the CGE Model

The estimates of elasticity presented in the table 6.4 have been used to perform a

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<sup>24</sup> Whereas a calculation of wage in this study should not be affected since a typical adjustment for self-employed income assumes that each self-employed worker is paid an average wage.

simulation exercise in the Scottish Government’s single-region CGE model. Currently, the elasticity value used across all sectors in the CGE model is 0.3. For this simulation exercise, the value of 0.3 has been changed in each sector to reflect new elasticities and a 5% negative labour supply shock has been introduced. On impact, the negative labour shock causes real wages to go up. Under perfect competition, factor prices are equal to their marginal products. Recall that elasticity in the CES production frame-work is defined as:

$$\sigma = \frac{d \ln(\frac{K}{L})}{d \ln(\frac{w}{r})} \quad (6.1)$$

To keep  $\sigma$  constant, when the denominator rises, the numerator has to go up. Therefore, cost-minimising firms should substitute away from a more expensive input, in this case labour, to a relatively less expensive input; capital. The rate of substitution between factors is governed by the elasticity parameter in each industry. In summary, we expect an increase in capital-intensity,  $K/L$ , ceteris paribus, unless factors are perfect complements. The results of the shock, using our new elasticities, show the capital stock declining by less and employment declining by more in comparison with the old model (see Figure 9.1 in Appendix 6). This provides some evidence in support of higher capital intensity, which is also increasing in the value of elasticity.

## 7. Conclusions

This dissertation produced estimates of the elasticity of substitution for the aggregate UK economy and individual industries using UK data. I used both single and multiple equation estimation frameworks and find that elasticity estimates are highly sensitive to the choice of the method.

Using single estimation techniques, such as OLS and FD, produced aggregate elasticity estimates in the region of 0.5. However, using the system estimation method of NLSUR produced estimates in the region of 0.82-1.06. System methods are thought to be superior as they deliver a more efficient estimation; they also alleviate simultaneity bias which is inherent in single equation techniques. To account for the remaining endogeneity, I used a 3SLS approach where the endogenous right-hand side variables are instrumented by their lags. This results in the estimate of 0.94 for the UK.

However, this system estimation approach comes with a limitation as it is sensitive to the initial values of parameters selected. In order to better identify global optima, a more thorough grid search across a wide range of initial values should be undertaken. This, along with the application of an optimisation algorithm that accounts for discontinuities in the CES function, should strengthen these results.

The elasticity estimates presented here are all higher than the 0.3 currently used in the Scottish Governments CGE model. They are also higher than previous estimates for the UK based on the user cost of capital equations which are in the neighbourhood of 0.4. Nevertheless, my estimates are comparable to those obtained in US studies which use similar system estimation approaches. In addition, my estimates across broader sectors such as manufacturing and agriculture are consistent with previous industry/sectoral level studies.

I also estimate elasticity for the aggregate economy of Scotland (1.3) and find it to be higher than my results for the UK. Sectoral estimates of elasticity in Scotland are also somewhat higher relative to the UK but these results should be interpreted with caution as they are likely to suffer from a small sample bias and may reflect poorer quality of data.

Along with elasticity, I estimate technical growth parameters. In line with previous studies, I conclude that elasticity estimates are biased upwards under neutral technical progress. I find that technological progress is overall labour-augmenting in the UK which supports the neoclassical steady-state growth theorem. Nevertheless, I also see robust evidence of capital-augmenting growth in some industries. With augmentation directed at labour, and under complementarity of capital and labour, I conclude that technical change is capital-biased in the aggregate economy and in the manufacturing sector, which is consistent with the declining labour share of income observed in the data.

Further research could focus on disaggregation of labour and capital input by skills and asset type, and introduction of a nesting structure to the CES production function where output is produced by routine and non-routine inputs. In this framework, an elasticity of substitution between unskilled workers and ICT capital (as a measure of automation) can be estimated at industry/sectoral level.

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